

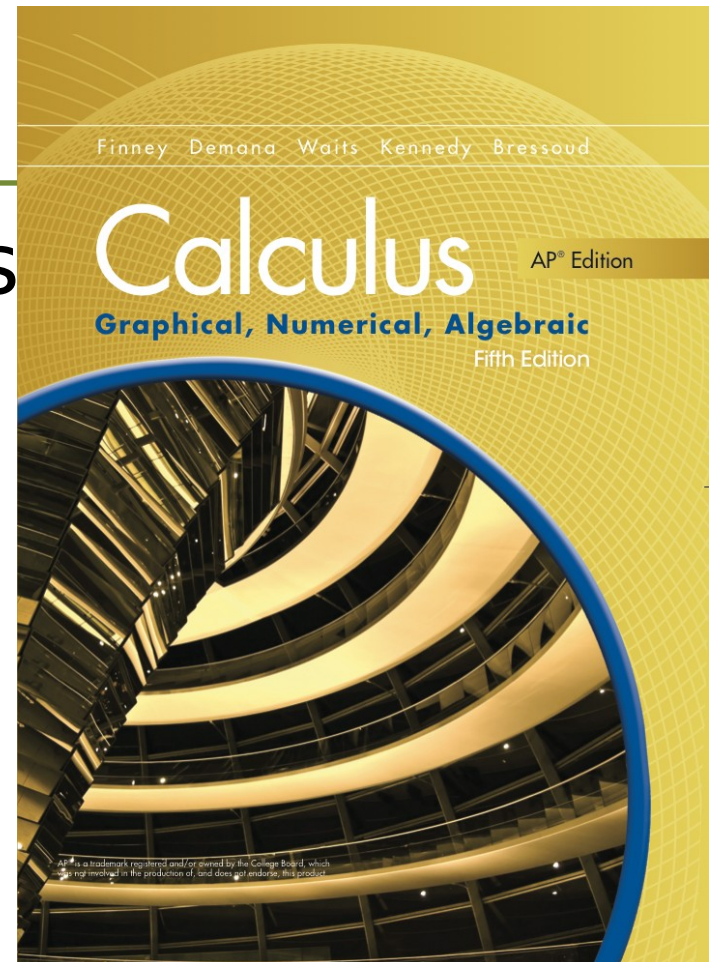
Chapter 1

Prerequisites for Calculus



Section 1.6

Trigonometric Functions



Quick Review

In Exercises 1 – 4, convert from radians to degrees or degrees to radians.

1. $\frac{\pi}{3}$

2. -2.5

3. -40°

4. 45°

Quick Review

In Exercises 5- 7, solve the equation graphically in the given interval.

5. $\sin x = 0.6$, $0 \leq x < 2\pi$

6. $\cos x = -0.4$, $0 \leq x < 2\pi$

7. $\tan x = 1$, $-\frac{\pi}{2} \leq x < \frac{3\pi}{2}$

Quick Review

8. Show that $f(x) = 2x^2 - 3$ is an even function.

Explain why its graph is symmetric about the y -axis.

9. Show that $f(x) = x^3 - 3x$ is an odd function.

Explain why its graph is symmetric about the origin.

10. Give one way to restrict the domain of the function $f(x) = x^4 - 2$ to make the resulting function one-to-one.

Quick Review

In Exercises 1–4, convert from radians to degrees or degrees to radians.

1. $\frac{\pi}{3}$ 60°
2. -2.5 -143.24°
3. -40° $-\frac{2\pi}{9}$
4. 45° $\frac{\pi}{4}$

Quick Review

In Exercises 5 – 7, solve the equation graphically in the given interval.

5. $\sin x = 0.6$, $0 \leq x < 2\pi$

$x \approx 0.6435, 2.4981$

6. $\cos x = -0.4$, $0 \leq x < 2\pi$

$x \approx 1.9823, 4.3009$

7. $\tan x = 1$, $-\frac{\pi}{2} \leq x < \frac{3\pi}{2}$

$x \approx 0.7854, 3.9270$

Quick Review

8. Show that $f(x) = 2x^2 - 3$ is an even function.

Explain why its graph is symmetric about the y -axis.

$$f(-x) = 2(-x)^2 - 3 = 2x^2 - 3 = f(x)$$

The graph is symmetric about the y -axis because if a point (a, b) is on the graph, then so is the point $(-a, b)$.

Quick Review

9. Show that $f(x) = x^3 - 3x$ is an odd function.

Explain why its graph is symmetric about the origin.

$$f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -f(x)$$

The graph is symmetric about the origin because if a point (a, b) is on the graph, then so is the point $(-a, -b)$.

Quick Review

10. Give one way to restrict the domain of the function $f(x) = x^4 - 2$ to make the resulting function one-to-one.

$x \geq 0$

What you'll learn about

- Radian measure
- The six basic trigonometric functions
- Periodicity
- Properties of trigonometric functions (symmetry, period)
- Transformations of trigonometric functions
- Sinusoids and their properties (amplitude, period, frequency, shifts)
- Inverse trigonometric functions and their graphs

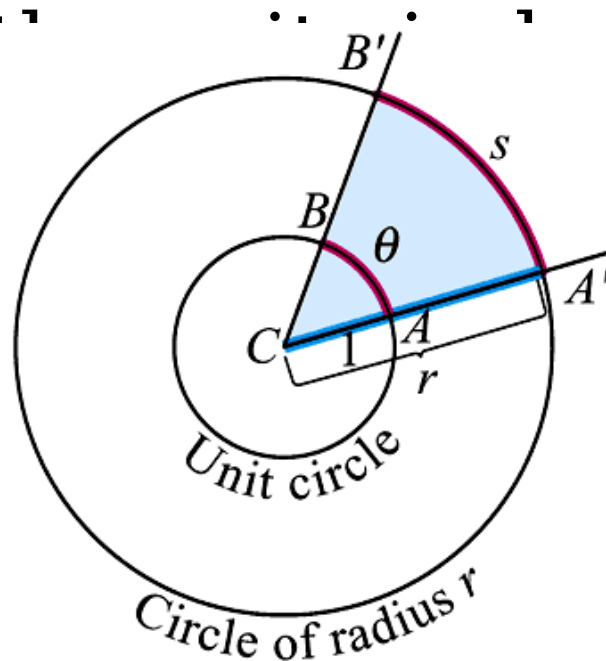
What you'll learn about

...and why

Trigonometric functions can be used to model periodic behavior and applications such as musical notes.

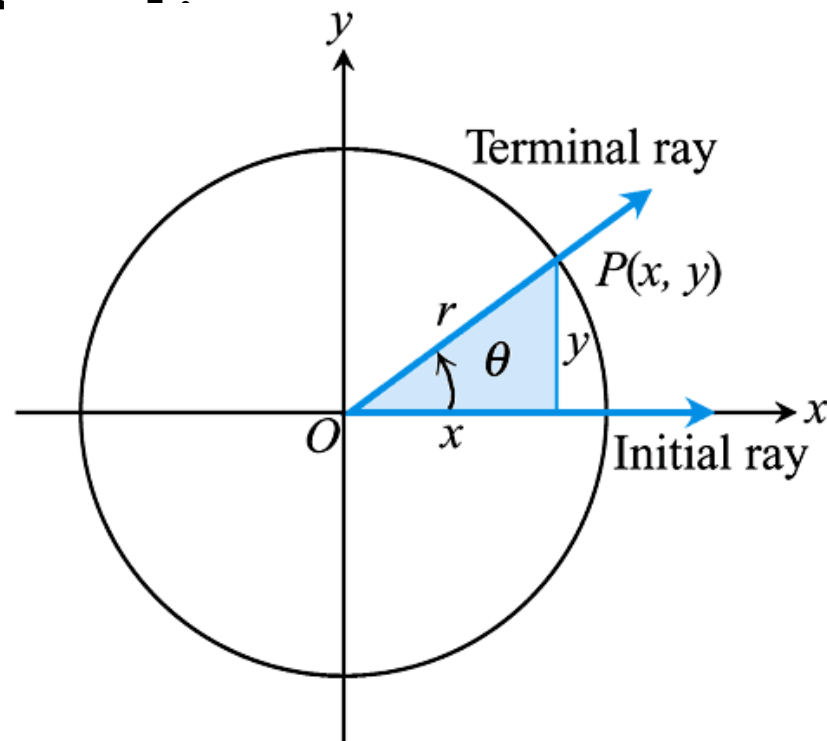
Radian Measure

- The **radian measure** of the angle ACB at the center of the unit circle equals the length of the arc that ACB cuts from the circle.



Radian Measure

- An angle of measure θ is placed in standard position at the center of circle of



Trigonometric Functions of θ

The six basic trigonometric functions of θ are defined as follows:

$$\text{sine: } \sin \theta = \frac{y}{r}$$

$$\text{cosecant: } \csc \theta = \frac{r}{y}$$

$$\text{cosine: } \cos \theta = \frac{x}{r}$$

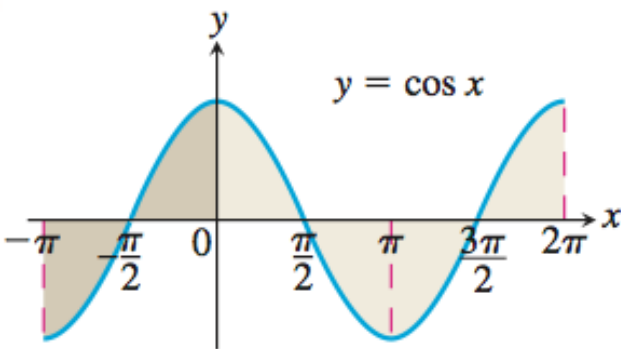
$$\text{secant: } \sec \theta = \frac{r}{x}$$

$$\text{tangent: } \tan \theta = \frac{y}{x}$$

$$\text{cotangent: } \cot \theta = \frac{x}{y}$$

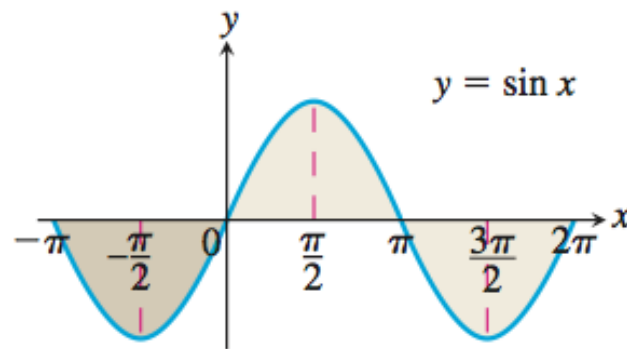
Graphs of Trigonometric Functions

When we graph trigonometric functions in the coordinate plane, we usually denote the independent variable (radians) by x instead of θ .



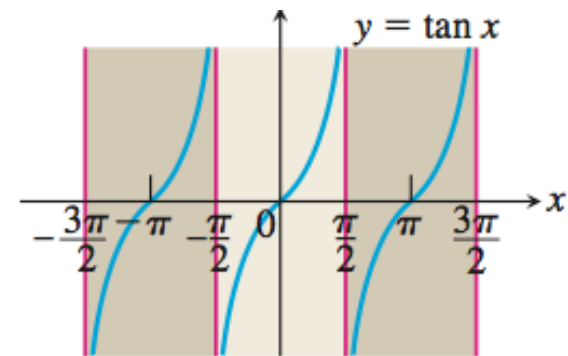
Domain: $-\infty < x < \infty$
 Range: $-1 \leq y \leq 1$
 Period: 2π

(a)



Domain: $-\infty < x < \infty$
 Range: $-1 \leq y \leq 1$
 Period: 2π

(b)

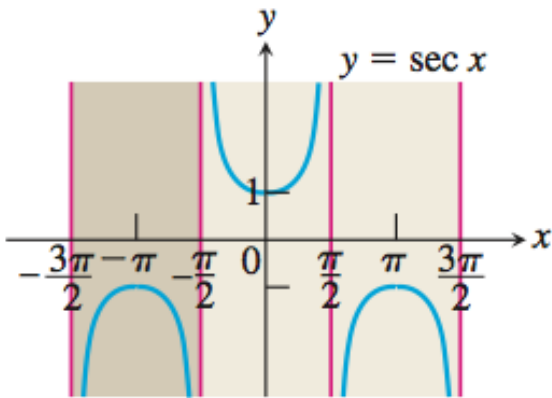


Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$
 Range: $-\infty < y < \infty$
 Period: π

(c)

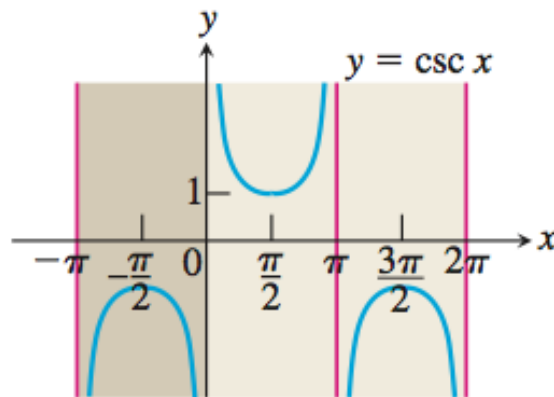
Graphs of Trigonometric Functions

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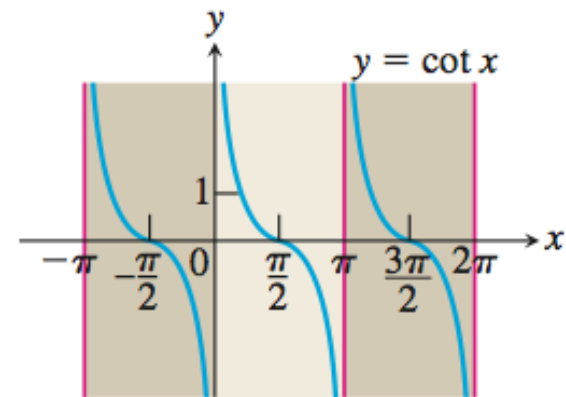
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$
 Range: $y \leq -1$ and $y \geq 1$
 Period: 2π

(d)



Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$
 Range: $y \leq -1$ and $y \geq 1$
 Period: 2π

(e)



Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$
 Range: $-\infty < y < \infty$
 Period: π

(f)

Angle Convention: Use Radians

From now on in this book, it is assumed that all angles are measured in radians unless degrees or some other unit is stated explicitly. When we talk about the angle $\frac{\pi}{3}$ we mean $\frac{\pi}{3}$ radians (which is 60°), not $\frac{\pi}{3}$ degrees. When you do calculus, keep your calculator in radian mode.

Periodic Function, Period

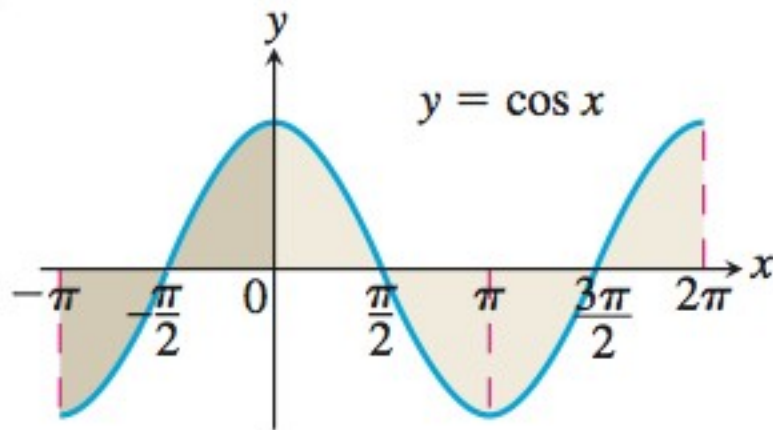
A function $f(x)$ is **periodic** if there is a positive number p such that $f(x + p) = f(x)$ for every value of x .

The smallest value of p is the **period** of f .

The functions $\cos x$, $\sin x$, $\sec x$ and $\csc x$ are periodic with period 2π . The functions $\tan x$ and $\cot x$ are periodic with period π .

Even Trigonometric Functions

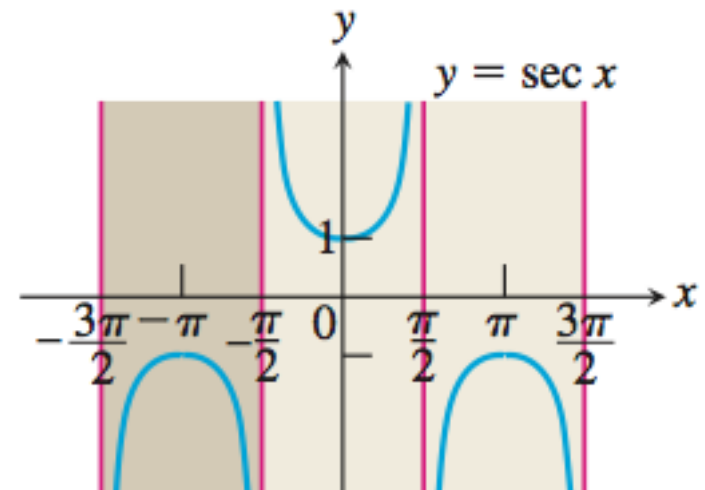
The graphs of $\cos x$ and $\sec x$ are **even** functions because their graphs are symmetric about the y -axis.



Domain: $-\infty < x < \infty$

Range: $-1 \leq y \leq 1$

Period: 2π



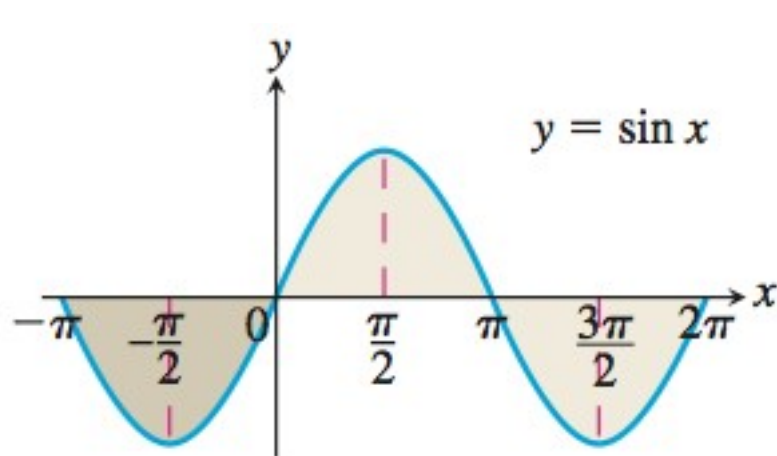
Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$

Range: $y \leq -1$ and $y \geq 1$

Period: 2π

Odd Trigonometric Functions

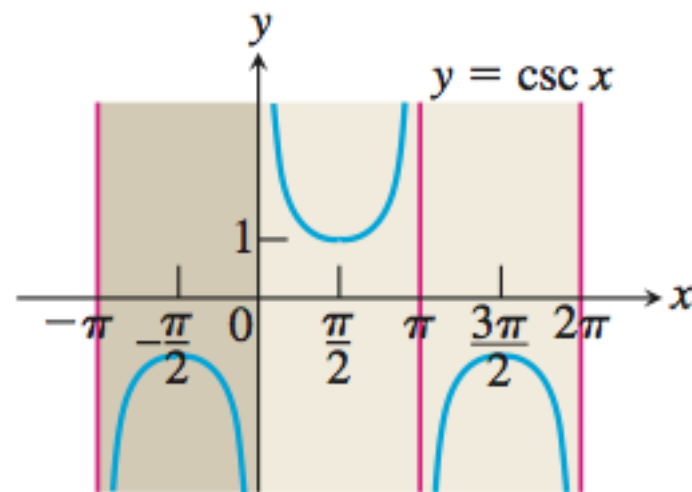
The graphs of $\sin x$, $\csc x$, $\tan x$ and $\cot x$ are **odd** functions.



Domain: $-\infty < x < \infty$

Range: $-1 \leq y \leq 1$

Period: 2π



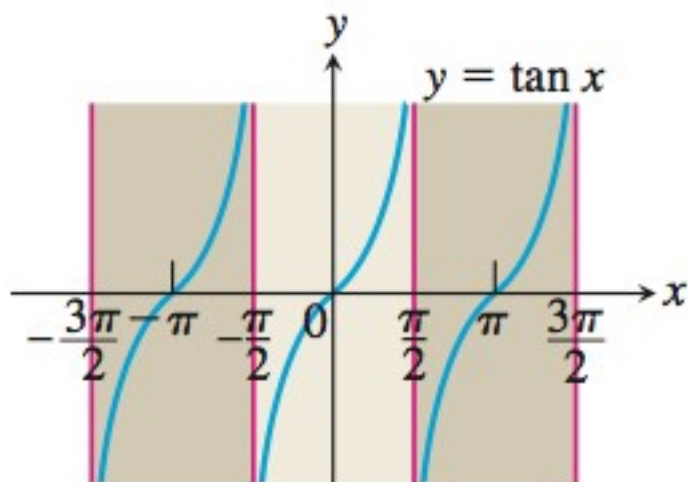
Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$

Range: $y \leq -1$ and $y \geq 1$

Period: 2π

Odd Trigonometric Functions

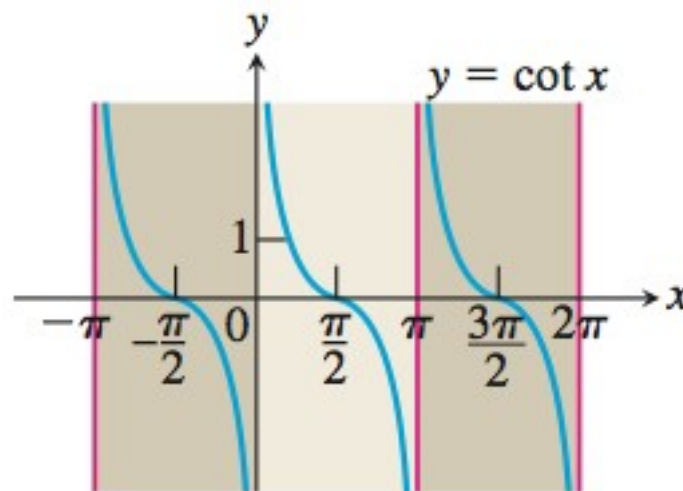
The graphs of $\sin x$, $\csc x$, $\tan x$ and $\cot x$ are **odd** functions.



Domain: $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$

Range: $-\infty < y < \infty$

Period: π



Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$

Range: $-\infty < y < \infty$

Period: π

Example Even and Odd Trigonometric Functions

Show that $\csc x$ is an odd function.

$$\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin x} = -\csc x$$

Transformations of Trigonometric Graphs

- The rules for shifting, stretching, shrinking and reflecting the graph of a function apply to the trigonometric functions.

Vertical stretch or shrink

Reflection about x-axis

$$y = a f(b(x + c)) + d$$

Vertical shift

Horizontal stretch or shrink

Horizontal shift

Reflection about the y-axis

Example

Transformations of Trigonometric Graphs

Determine the period, domain, range and draw the graph of

$$y = -2\sin(4x + \pi).$$

Example

Transformations of Trigonometric Graphs

Rewrite the function as $y = -2 \sin\left(4\left(x + \frac{\pi}{4}\right)\right)$

The period of $y = a \sin bx$ is $\frac{2\pi}{b}$.

$b=4$, so the period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

The domain is $(-\infty, \infty)$.

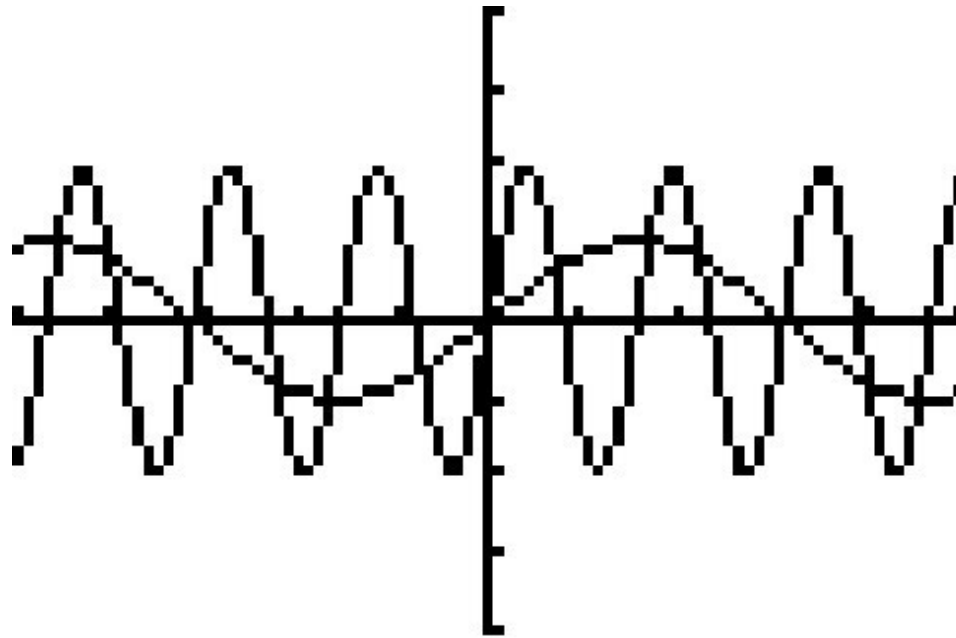
The graph is a basic $\sin x$ curve with an amplitude of 2.

Thus, the range is $[-2, 2]$.

Example

Transformations of Trigonometric Graphs

The graph of the function is shown together with the graph of the $\sin x$ function.



$[-5, 5]$ by $[-4, 4]$

Inverse Trigonometric Functions

- None of the six basic trigonometric functions graphed on slides 15 and 16 is one-to-one. These functions do not have inverses. However, in each case, the domain can be restricted to produce a new function that does have an inverse.
- The domains and ranges of the inverse trigonometric functions become part of their definitions.

Function

Domain

Range

$$y = \cos^{-1} x$$

$$-1 \leq x \leq 1$$

$$0 \leq y \leq \pi$$

$$y = \sin^{-1} x$$

$$-1 \leq x \leq 1$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \tan^{-1} x$$

$$-\infty < x < \infty$$

$$-\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$y = \sec^{-1} x$$

$$|x| \geq 1$$

$$0 \leq y \leq \pi, y \neq \frac{\pi}{2}$$

$$y = \csc^{-1} x$$

$$|x| \geq 1$$

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$$

$$y = \cot^{-1} x$$

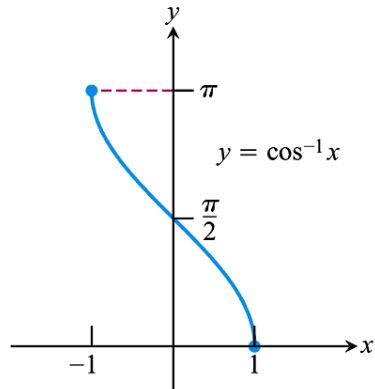
$$-\infty < x < \infty$$

$$0 < y < \pi$$

Inverse Trigonometric Functions

Domain: $-1 \leq x \leq 1$

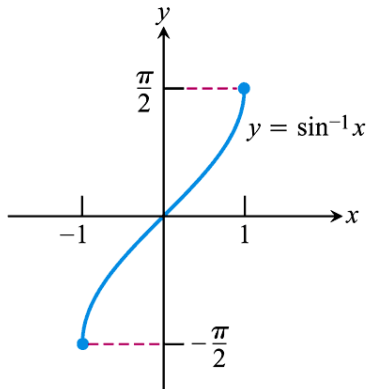
Range: $0 \leq y \leq \pi$



(a)

Domain: $-1 \leq x \leq 1$

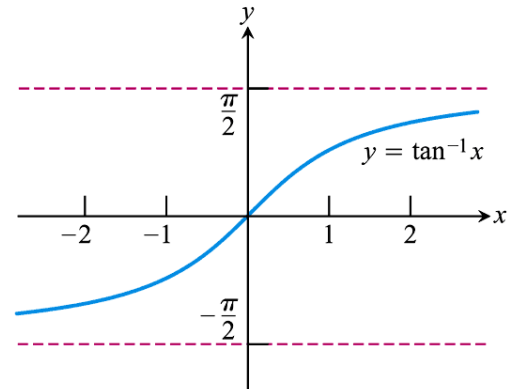
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



(b)

Domain: $-\infty < x < \infty$

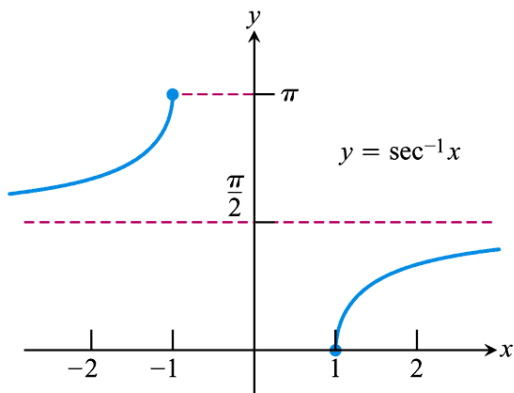
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



(c)

Domain: $x \leq -1$ or $x \geq 1$

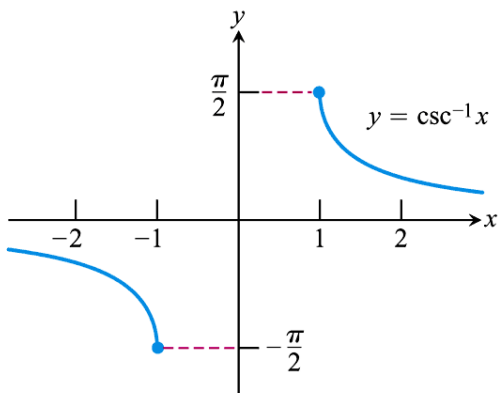
Range: $0 \leq y \leq \pi, y \neq \frac{\pi}{2}$



(d)

Domain: $x \leq -1$ or $x \geq 1$

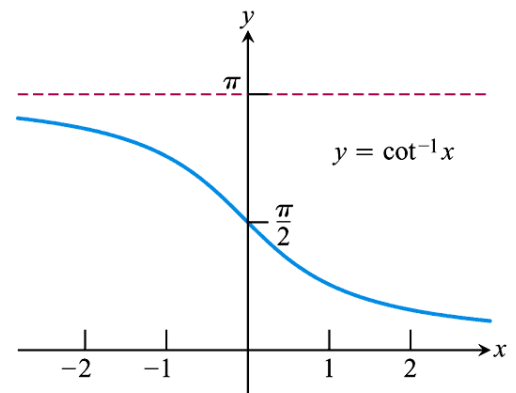
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$



(e)

Domain: $-\infty < x < \infty$

Range: $0 < y < \pi$



(f)

Example Inverse Trigonometric Functions

Find the measure of $\sin^{-1}\left(-\frac{1}{2}\right)$ in degrees.

Put the calculator in degree mode and enter $\sin^{-1}\left(-\frac{1}{2}\right)$.

The calculator returns -30° .

Example Inverse Trigonometric Functions

Find the measure of $\sin^{-1}\left(-\frac{1}{2}\right)$ in radians.

Put the calculator in radian mode and enter $\sin^{-1}\left(-\frac{1}{2}\right)$.

The calculator returns - .52359877556 radians.

This is the same as $-\frac{\pi}{6}$ radians.

Quick Quiz Sections 1.4

1.6

You should solve the following problems without using a graphing calculator.

1. Which of the following is the domain of

$$f(x) = -\log_2(x + 3)?$$

(A) $(-\infty, \infty)$

(B) $(-\infty, 3)$

(C) $(-3, \infty)$

(D) $[-3, \infty)$

(E) $(-\infty, 3]$

Quick Quiz Sections 1.4

-1.6

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(B) $(-\infty, 3)$

(C) $(-3, \infty)$

(D) $[-3, \infty)$

(E) $(-\infty, 3]$

Quick Quiz Sections 1.4 – 1.6

2. Which of the following is the range of

$$f(x) = 5\cos(x + \pi) + 3?$$

(A) $(-\infty, \infty)$

(B) $[2, 4]$

(C) $[-8, 2]$

(D) $[-2, 8]$

(E) $\left[-\frac{2}{5}, \frac{8}{5}\right]$

Quick Quiz Sections 1.4 – 1.6

2. Which of the following is the range of

$$f(x) = 5\cos(x + \pi) + 3?$$

(A) $(-\infty, \infty)$

(B) $[2, 4]$

(C) $[-8, 2]$

(D) $[-2, 8]$

(E) $\left[-\frac{2}{5}, \frac{8}{5}\right]$

Quick Quiz Sections 1.4 – 1.6

3. Which of the following gives the solution of

$$\tan x = -1 \text{ in } \pi \leq x \leq \frac{3\pi}{2}?$$

(A) $-\frac{\pi}{4}$

(D) $\frac{3\pi}{4}$

(B) $\frac{\pi}{4}$

(E) $\frac{5\pi}{4}$

(C) $\frac{\pi}{3}$

Quick Quiz Sections 1.4 – 1.6

3. Which of the following gives the solution of

$$\tan x = -1 \text{ in } \pi \leq x \leq \frac{3\pi}{2}?$$

(A) $-\frac{\pi}{4}$

(D) $\frac{3\pi}{4}$

(B) $\frac{\pi}{4}$

(E) $\frac{5\pi}{4}$

(C) $\frac{\pi}{3}$

Chapter Test

In Exercises 1 and 2, write an equation for the specified line.

1. through $(4, -12)$ and parallel to $4x + 3y = 12$

2. the line $y = f(x)$ where f has the following values:

x	-2	2	4
$f(x)$	4	2	1

Chapter Test

3. Determine whether the graph of the function

$y = x^{\frac{1}{5}}$ is symmetric about the y -axis, the origin, or neither.

4. Determine whether the function $y = \frac{x^4 + 1}{x^3 - 2x}$

is even, odd or neither.

Chapter Test

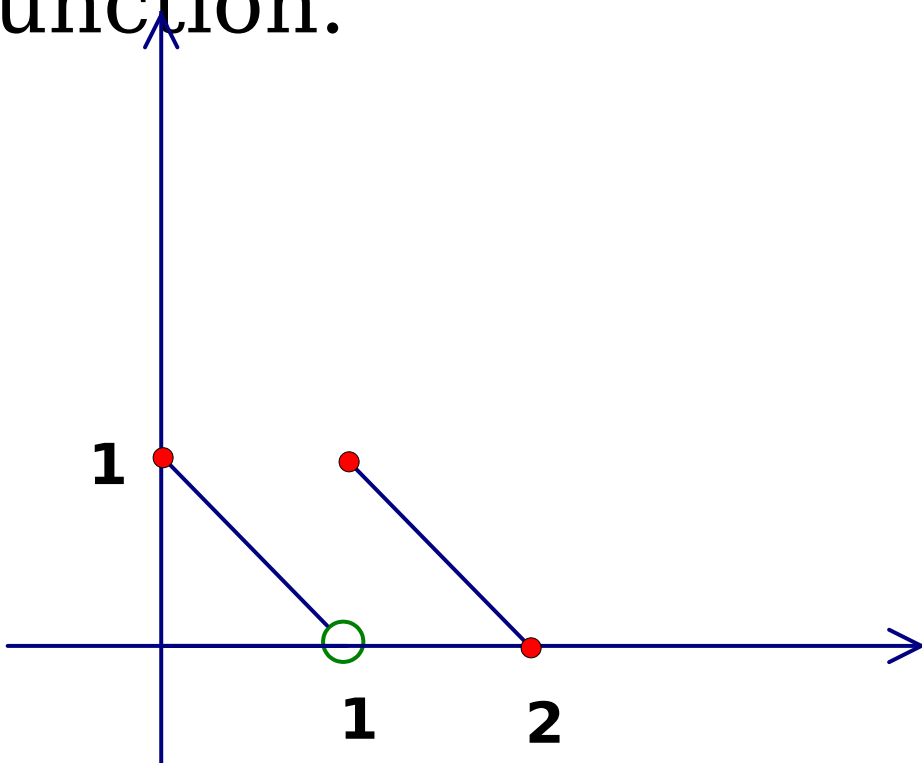
In Exercises 5 and 6, find the (a) domain and (b) range, and (c) graph the function.

5. $y = 2\sin(3x + \pi) - 1$

6. $y = \ln(x - 3) + 1$

Chapter Test

7. Write a piecewise formula for the function.



Chapter Test

8. $x=5\cos t$, $y=2\sin t$, $0 \leq t \leq 2\pi$ is a parametrization for a curve.

(a) Graph the curve.

Identify the initial and terminal points, if any.

Indicate the direction in which the curve is traced.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

Chapter Test

9. Give one parametrization for the line segment with endpoints $(-2, 5)$ and $(4, 3)$.
10. Given $f(x) = 2 - 3x$,
- (a) find f^{-1} and show that
$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$$
 - (b) graph f and f^{-1} in the same viewing window

Chapter Test Solutions

In Exercises 1 and 2, write an equation for the specified line.

1. through $(4, -12)$ and parallel to $4x + 3y = 12$

$$y = -\frac{4}{3}x - \frac{20}{3}$$

2. the line $y = f(x)$ where f has the following values:

x	-2	2	4
$f(x)$	4	2	1

$$y = -\frac{1}{2}x + 3$$

Chapter Test Solutions

3. Determine whether the graph of the function

$y = x^{\frac{1}{5}}$ is symmetric about the y -axis, the origin, or neither.

Origin

4. Determine whether the function $y = \frac{x^4 + 1}{x^3 - 2x}$

is even, odd or neither.

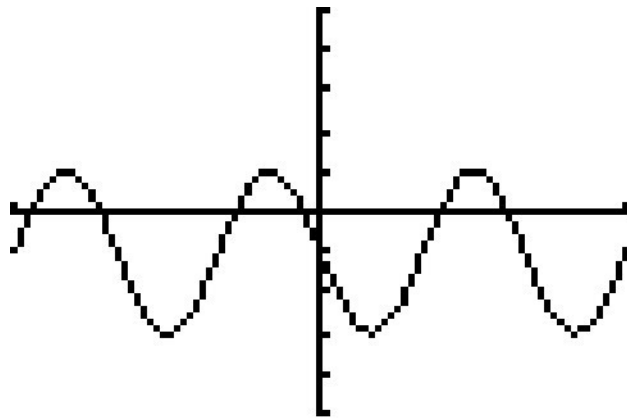
Odd

Chapter Test Solutions

In Exercises 5 and 6, find the (a) domain and (b) range, and (c) graph the function.

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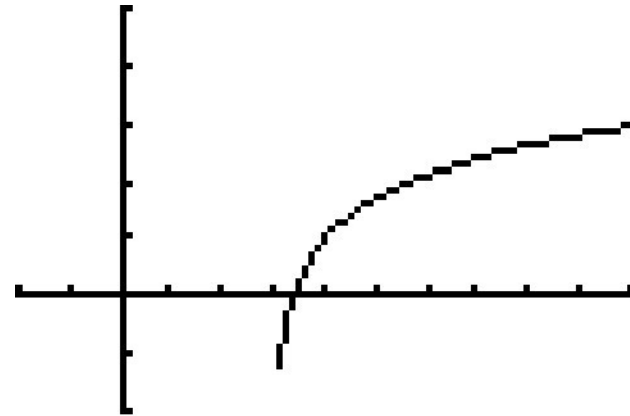
(a) All Reals (b) $[-3, 1]$



$[-\pi, \pi]$ by $[-5, 5]$

6. $y = \ln(x - 3) + 1$

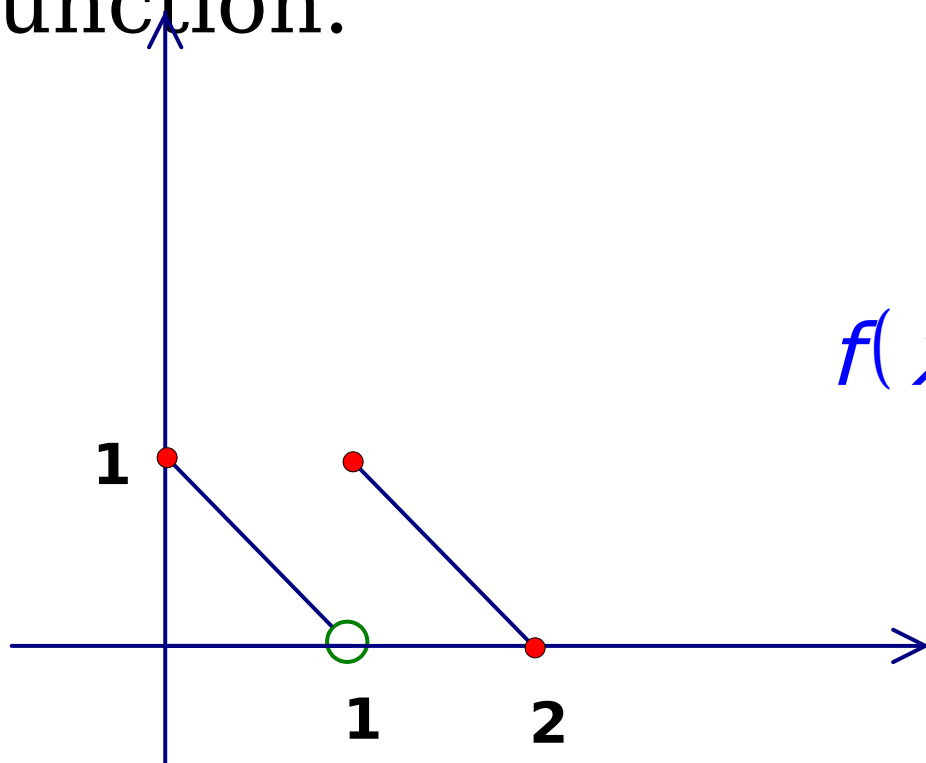
(a) $(3, \infty)$ (b) All Reals



$[-2, 10]$ by $[-2, 5]$

Chapter Test Solutions

7. Write a piecewise formula for the function.



$$f(x) = \begin{cases} 1 - x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$$

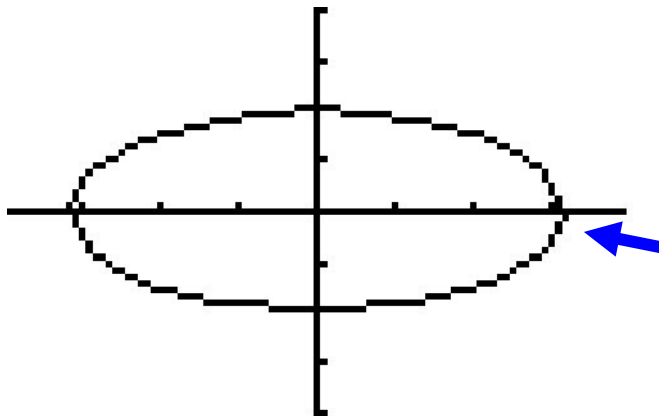
Chapter Test Solutions

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(a) Graph the curve.

Identify the initial and terminal points, if any.

Indicate the direction in which the curve is traced.



Initial Point (5, 0)
Terminal Point (5, 0)

Chapter Test Solutions

8. $x=5\cos t$, $y=2\sin t$, $0 \leq t \leq 2\pi$ is a parametrization for a curve.

(b) Find a Cartesian equation for a curve that contains the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1, \quad \text{All}$$

Chapter Test Solutions

9. Give one parametrization for the line segment with endpoints $(-2, 5)$ and $(4, 3)$.

(other answers are possible)

$$x = -2 + 6t, \quad y = 5 - 2t, \quad 0 \leq t \leq 1$$

Chapter Test Solutions

10. Given $f(x) = 2 - 3x$,

(a) find f^{-1} and show that

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$$

$$f^{-1}(x) = \frac{2 - x}{3}$$

$$f(f^{-1}(x)) = f\left(\frac{2 - x}{3}\right) = 2 - 3\left(\frac{2 - x}{3}\right) = 2 - (2 - x) = x$$

$$f^{-1}(f(x)) = f^{-1}(2 - 3x) = \frac{2 - (2 - 3x)}{3} = \frac{3x}{3} = x$$

Chapter Test Solutions

(b) graph f and f^{-1} in the same viewing window

